A Comparative Analysis of the Center of Gravity and Center of Pressure Trajectory Path Lengths in Standing Posture: An Estimation of Active Stiffness

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The center of foot pressure (CP) motions, representing the net neuromuscular control, was compared to the center of gravity (CG) motions, representing the net performance. The comparison focused on the trajectory path length parameter along the medio-lateral and antero-posterior axes because these two variables depend on amplitude versus frequency relationship. This relationship was used to evaluate the CG motions based on the CP motions. Seven subjects stood still on a force plate with eyes open and eyes closed. The results showed that the ratio of (CP – CG)/CP trajectory path length was personal for each subject. These results suggest different levels of passive (ligaments, elastic properties) and active (reflex activity) stiffness. For some subjects, this ratio was significantly lower for the eyes open condition than for the eyes closed condition, indicating a decrease of the active stiffness for the eyes open condition. Therefore, a CG – CP comparative analysis appeared helpful in understanding the control of balance and necessary to quantify the subjects’ net performance.

Key Words: center of gravity, center of pressure, trajectory path length, time/domain transformations, stiffness, standing posture

Nomenclature

CG center of gravity
CP center of pressure
x medio-lateral axis
y antero-posterior axis
z vertical axis
→OM angular momentum of the center of gravity
\( \sum F_{ext} \) moment of all the external forces
θ body inclination
\( \vec{γ}_g \) angular acceleration of the CG
\( \vec{R}_y \vec{R}_z \) reaction forces along the antero-posterior and vertical axes
m, g total body mass, gravity acceleration
\( \vec{P} = mg \) body weight
h distance from CG to the ground

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Postural Control and Balance Analysis

In standing posture, the aim of the task is usually to stand still. In order to evaluate the subjects’ performance, it would first need to be quantified. It is generally assumed that the body is relatively rigid and oscillates as a one-link inverted pendulum with the rotation axis at the ankle (Gurfinkel & Osovets, 1972; Nashner, 1971). Based on this assumption, the center of gravity (CG) motion appears to be a satisfactory parameter to evaluate the subjects’ oscillations or net performance.

Until now, the CG motion has rarely been evaluated in standing posture (Day, Steiger, Thompson, & Marsden, 1993; Hasan et al., 1996; Winter, 1990; Winter, Prince, Stergiou, & Powel, 1993; Winter et al., 1996). The main reason was probably the difficulty in evaluating this CG trajectory. Therefore, the standing analysis is often based on the center of foot pressure (CP) motion, defined as “the net neuromuscular response to control of the passive CG” (Winter et al., 1996). This CP motion oscillates to either side of the CG motion with higher amplitude and higher frequency (Thomas & Whitney, 1959; Winter, 1990). Recently, Brenière (1996) established an amplitude CG/CP relationship in the frequency domain. Consequently, a simple and precise method to evaluate the CG horizontal positions based on the CP horizontal positions was proposed (Caron, Faure, & Brenière, 1997) and is used here.

To compare the subjects’ CG and CP motions, it is possible to use a large number of parameters (trajectory path length, mean distance from the body sway center, area of an ellipse; see Fitzgerald, Murray, Elliott, & Birchall, 1994; Geurts, Nienhuis, & Mulder, 1993). Nevertheless, on the basis of the amplitude CG/CP relationship in the frequency domain presented below, there is a linear relationship between the CP and CG values for parameters calculating an average of the CP and CG horizontal positions (such as the area of an ellipse, the standard deviation; Caron, 1997). On the contrary, the relationship between the CP and CG values is less pronounced for parameters that take into account all the CP and CG horizontal positions (Caron, 1997)—in other words, for parameters that are sensitive to both amplitude and frequency modifications as the trajectory path length. Therefore, for this parameter, the hypothesis is that a similar CG trajectory path length could be achieved with different CP trajectory path lengths. The aim of this study is to compare the subjects’ CG and CP trajectory path lengths, and the significance of this comparative analysis is discussed.
Methods

The standing behavior of seven healthy male subjects was investigated (age: 27.6 years ± 4.2; height: 181.9 cm ± 5.5; mass: 79.3 kg ± 5.1). All the subjects gave their informed consent.

Two conditions were carried out, with each foot positioned to form a 15° angle relative to the antero-posterior axis, heels 4-cm apart, and arms loosely hanging at the sides. The subjects were instructed to stand as still as possible with eyes open (EO) and with eyes closed (EC) at random. They performed five trials of 64 s for each condition.

A force plate, composed of a steel plate supported by three monoaxial load cells (Schlumberger, model CD-750), was used to compute the CP horizontal positions. The signals from the force transducers were amplified (PM, model 1965) and recorded (Mazet Electronique, model Biostim 6082) at 64 Hz during 64 s.

The CG horizontal positions were evaluated on the basis of the CP horizontal positions (Caron, Faure, & Brenière, 1997), using a low-pass filter defined by a mathematical relationship of the relative magnitude of the CG with respect to the magnitude of the CP, as a function of the frequency (Brenière, 1996). This relationship was computed from the derivative of the angular momentum equation applied to the whole body with respect to the CG. In this equation, the rotations around the vertical axis are neglected, and the moment of inertia and the height of the CG from the ground are considered to be constant. The equation can be written:

\[ \frac{d(\sigma^\rightarrow_g)}{dt} = \Sigma M^\rightarrow_{Fext} \]  

(1)

In standing, it is possible to consider that the angular oscillations correspond to the CG horizontal motions. It can be written for the sagittal plane:

\[ -h\theta(t) = y_g(t) \] and after two derivations \[ \ddot{y}_g(t) = -\dot{\theta}(t) / h \]  

(2)

On the basis of Newton’s second law, we can write the following equations:

\[ R_y = m\ddot{y}_g \]  

(3)

\[ R_z = m\ddot{z}_g + mg \]  

(4)

In order to define the amplitude CG/CP relationship in the frequency domain, the CP and CG horizontal oscillations are considered as simple periodic functions. For a given pulsation (angular frequency) and for the sagittal plane, it can be written:

\[ y_g(t) = [y_g] \cos(\Omega t) \] and after two derivations \[ \ddot{y}_g(t) = -\Omega^2 y_g(t) \]  

(5)

Therefore, on the basis of the Figure 1 and using the equation 1, it can be written for the sagittal plane:

\[ I_{Gx} \ddot{y}_g = \dot{z}_g \times R_y + \dot{d} \times R_z \] or \[ I_{Gx} \ddot{y}_g = -(\dot{y}_g)R_y + (y_p - y_g)R_z \]  

(6)

by using the equations 1, 2, and 3, and by multiplying the equation by h, equation 6 becomes:

\[ -\ddot{y}_g (mh^2 + I_{Gx}) = mgh(y_p - y_g)(z_g / g + 1) \]  

(7)

as \( z_g \) is assumed to be constant and equal to h, the term \( z_g / g \) is neglected. Equation 7 becomes:

\[ -\ddot{y}_g = \frac{mgh}{(mh^2 + I_{Gx})} (y_p - y_g) \]  

(8)
If we pose \( \Omega_0^2 = \frac{mgh}{(mh^2 + I_G)} \), equation 8 becomes:

\[
-\ddot{y}_g = \Omega_0^2 y_p - \Omega_0^2 y_g
\]  

(9)

And finally, by using the equation 5 we obtain:

\[
y_g / y_p = \Omega_0^2 / (\Omega^2 + \Omega_0^2)
\]  

(10)

This equation determines the amplitude CG/CP relationship in the frequency domain for the sagittal plane (Brenière, 1996). The same analysis can be done for the frontal plane. This model takes into account the biomechanical constants of the subjects defined by the term \( \Omega_0 \). This term takes into account the mass, the height and the distribution of this mass from the CG. For the present study, \( \Omega_0 \) is equal to 3.2 rad/s (Caron, Faure, & Brenière, 1997).
Specifically, a discrete fast Fourier transformation was computed on the CP positions to obtain the frequency spectrum of the CP motion. This frequency spectrum was multiplied by the mathematical relationship presented above to compute the evaluated frequency spectrum of the CG motion. Finally, an inverse Fourier transformation brought the data back to the time domain, defining the evaluated CG horizontal positions (Figure 3). After the CG horizontal positions have been evaluated, the CP positions were filtered using an ideal low-pass filter (i.e., with a gain equal to 1 or 0) with frequency cut-off at 3 Hz (Caron, Faure, & Brenière, 1997).

The accuracy of the method has previously been established (Caron, Faure, & Brenière, 1997) by comparing the CG horizontal accelerations measured with a force plate (AMTI, model OR6-5-1) and the CG horizontal accelerations derived from the estimated CG positions. The average root mean square difference between these two accelerations was very small (<0.01 m/s²) and inferior to 6% to the standard deviation of the CG horizontal accelerations measured with the force plate.

This method, with reasonable approximation, can be considered independent of the biomechanical constants because \( \Omega_0^2 \) is present throughout the quotient of the amplitude CG/CP ratio. This independence was experimentally confirmed (Caron, Faure, & Brenière, 1997).

The trajectory path length for each trial was computed for the CP and CG motions (i.e., the total displacement during each 62-s trial period; first and last seconds were deleted due to signal processing) along the medio-lateral and antero-posterior axes. For each trial, a CG/CP ratio of the trajectory path length was computed as follows: \( R = \frac{(\text{CP} - \text{CG})}{\text{CP}} \cdot 100. \)
Figure 3 — Representation of the CG and CP horizontal positions for two subjects’ trials (subject 4) for the eyes open (EO) and eyes closed (EC) conditions for the antero-posterior axis as a function of time (top). The average CG position is equal to 154 mm for the EO condition and to 155 mm for the EC condition, whereas the \((CP - CG)/CP\) ratio is equal to 45% and 55% for the EO and EC conditions, respectively. The magnitude spectrum of these curves (middle) and the magnitude difference between the CP and CG motions (bottom) are also represented. Note that this magnitude difference is greater for the EC condition than for the EO condition for the frequency lying between 0.25 and 0.35 Hz.
The values ranged from 0 (no difference between the CG and CP trajectory path lengths) to 100 (CG trajectory path length equal zero), with 50 indicating a trajectory path length two-fold longer for the CP than for the CG. In other words, the higher the \((CP - CG)/CP\) ratio, the greater the difference between the CG and CP trajectory path lengths.

Finally, the horizontal distance between the average CG positions and the middle of the two heels of each trial for the antero-posterior axis was computed.

The mean \((M)\) and standard deviation \((SD)\) values of the CG and CP trajectory path lengths, and the \((CP - CG)/CP\) ratio were computed for each subject. The comparison of means within each subject’s data between the two conditions for a given axis was performed using a Student’s \(t\) test. The level of significance chosen was \(p < .05\).

**Results**

For the two conditions and the two axes, the main result was that the CG and CP trajectory path lengths (TPL) followed different patterns (Figure 4). In other words, there is not a linear relationship between the subjects’ CG and CP TPL. As it was expected, for all of the subjects, the CG TPL was always shorter than the CP one.

The CG and CP TPL decreased from the EC to the EO condition. More precisely, the decrease of the CG TPL was significant for 4 subjects along the medio-lateral axis, and for 6 subjects along the antero-posterior axis (Figure 4). For the CP TPL, it was significant for 5 subjects along the medio-lateral axis and for 6 subjects along the antero-posterior axis.

For the lateral axis (Figure 5), the \((CP - CG)/CP\) ratio remained constant between the two conditions for 6 subjects, but was significantly greater for the EC condition for 1 subject (subject 4). For this subject, the CP TPL was significantly greater in the EC condition, whereas no significant difference was observed for the CG TPL between the two conditions (Figure 4).

For the antero-posterior axis (Figure 5), the \((CP - CG)/CP\) ratio remained constant between the two conditions for 5 subjects but was significantly greater for the EC condition for 2 subjects (subjects 4 and 5). For these 2 subjects, the CG and CP TPL were significantly greater in the EC condition.

Still for the antero-posterior-axis, for all of the subjects, no significant difference was observed for the average CG positions between the two conditions (Figure 6). For subjects 4 and 5, the difference of the average CG positions between the two conditions is equal to 3.7 mm and 0.7 mm, respectively, which corresponds approximately to an average modification of the inclination of 0.2° and 0.04°, respectively.

For subject 4, the magnitude spectrum of two trials along the antero-posterior axis (one for each condition) of the CP and CG horizontal positions is represented (Figure 3). As it was well described, the amplitude is lying between 0 and 0.5 Hz. The magnitude spectrum of the difference between CP and CG motions is also presented (Figure 3), indicating for this subject a greater magnitude for the EC condition for the frequency lying between 0.25 and 0.35 Hz.

**Discussion**

The aim of the present study was to compare the CG and CP trajectory path lengths representing the net performance and the net neuromuscular control respectively. The primary result was the different patterns observed for these two variables between the subjects for the eyes open (EO) and eyes closed (EC) conditions (Figures 4). These different patterns resulted in different \((CP - CG)/CP\) trajectory path length ratios between the subjects (Figure 5). These results were predictable because of the existence of the amplitude CG/CP...
Figure 4 — Means and standard deviations (SD) of the CG and CP trajectory path lengths (mm) along the medio-lateral (top) and antero-posterior axes (bottom) for the EC condition (left) and the EO condition (right) for the 7 subjects. The subjects’ CG trajectory path lengths are presented in decreasing order of performance (each number corresponds to a subject). Note that the CG and CP trajectory path lengths follow different patterns. The significant differences between the two conditions are indicated in the EO condition (right) by *$p < .05$ and **$p < .01$. Note that a significant difference for the CP trajectory path length is not always followed by a significant difference for the CG trajectory path length (subject 4, top; subject 6, bottom). The contrary is observed for subject 2 (bottom).
Figure 5 — Means and SD of the ratio of (CP – CG)/CP trajectory path length (mm) along the medio-lateral (top) and antero-posterior axes (bottom) for the two conditions. *$p < .05$ and **$p < .01$. Note that there is a significant difference between the two visual conditions (EO and EC) for 1 subject along the medio-lateral axis and for 2 subjects along the antero-posterior axis.

relationship in the frequency domain, recently established by Brenière (1996) and used to estimate the CG horizontal positions (Caron, Faure, & Brenière, 1997). This relationship (Figure 2) clearly shows that the amplitude of the CG motion depends on both amplitude and frequency CP motions. On the basis of this relationship, the higher the frequency of the CP oscillations, the greater the amplitude difference between the CG and the CP trajectory path lengths and, consequently, the higher the (CP – CG)/CP ratio. By contrast, the lower the frequency of the CP oscillations, the lower the amplitude difference between the two variables and the lower the (CP – CG)/CP ratio. Therefore, the different (CP – CG)/
The comparison of the CG and CP trajectory path lengths for each subject between the two conditions showed that these trajectory path lengths were always smaller for the EO condition (Figure 4). This decrease for the EO condition was significant for some of the subjects, and generally more pronounced for the antero-posterior axis than for the medio-lateral axis. The differences between these two axes would appear to result from the axis biomechanical differences and the different muscles involved. Winter et al. (1996) proposed that, in side by side stance, the antero-posterior balance is under ankle control, whereas the medio-lateral balance is under hip control.

Nevertheless, the (CP – CG)/CP ratio remained constant between the two conditions for 6 subjects along the medio-lateral axis and for 5 subjects along the antero-posterior axis (Figure 5). These results indicate that the decrease of the CP horizontal positions for the EO condition occurred in the very low frequency, as described previously (<0.1 Hz; Black, O’Leary, Wall, & Furman, 1977), producing a proportional decrease of the CG horizontal positions. However, this ratio was significantly lower in the EO condition for 1 subject along the medio-lateral axis and for 2 subjects along the antero-posterior axis (Figure 5). For these subjects, the decrease of the CP horizontal positions for the EO condition occurred, in part, for frequencies higher than 0.1 Hz, relatively increasing the CG horizontal positions and consequently producing a lower (CP – CG)/CP ratio (Figure 3).

For subject 4 (Figure 4), the significant decrease of the CP trajectory path length along the antero-posterior axis for the EO condition did not significantly lower the CG trajectory path length, confirming that the subjects’ net neuromuscular control cannot be used to evaluate the subjects’ net performance. This result was also observed for 2 subjects.
Caron, Gélat, Rougier, and Blanchi (subjects 2 and 6) along the antero-posterior axis (Figure 4). Moreover, a comparative analysis of two tasks resulting in a modification of the CP amplitude in high frequencies, such as the comparison between standing and sitting (Bouisset & Duchêne, 1994), will increase the differences between the subjects’ net neuromuscular control and net performance.

The differences observed between the CG and the CP trajectory path lengths may have several interpretations. They could be interpreted as an indicator of the subjects’ muscular efficiency; however, efficiency does not appear to be a relevant variable due to the low level of force developed by the muscles in standing posture (Okada, 1972).

Recently, Winter, Prince, and Patla (1997) have validated the inverted pendulum model of balance in quiet standing. When considering the inverted pendulum model, one must consider a restoring force. By definition, the natural pulsation (angular frequency) of an inverted pendulum is $\Omega_n = \sqrt{K_u / I}$ where $K_u$ is the torsional stiffness and $I$ the pendulum’s moment of inertia. For a subject, this torsional stiffness ($K_u$) depends on the active and passive stiffness. The first one is represented by the muscle stiffness and the second one by the elastic properties of the activated muscles, joints, and adjacent tissues in a given position. In standing, this position can be characterized by the angle between the body and the vertical axis, or by the horizontal distance between the CG and the heel, for example. By increasing this horizontal distance (or the angle), the subjects must raise their muscular force (concerning especially here the soleus) by increasing the recruitment of motor units, which is closely analogous to adding prestretched springs in parallel (Houk & Rymer, 1981). Consequently, recruitment of motor units produces an increase in mechanical stiffness. This mechanical stiffness can be defined as follows (Houk & Rymer, 1981):

$$K = \frac{\Delta f_m}{\Delta x}$$

where $f$ represents the muscle force and the subscript m designates that this is a purely mechanical response, and $\Delta x$ represents the muscle length. Therefore, an increase of the muscle stiffness will raise the torsional stiffness ($K_u$) of the inverted pendulum and consequently will increase the natural pulsation ($\Omega_n$) or frequency ($f_n$) of the inverted pendulum. As the CP horizontal positions represent the net neuromuscular control, an increase of the muscle stiffness will increase the frequency of the CP oscillations. Finally, on the basis of the amplitude CG/CP relationship in the frequency domain (Figure 2), the higher the frequency of the CP oscillations, the greater the amplitude difference between CP and CG motions (see Figure 3). Therefore, it is possible to say that the higher the stiffness, the greater the amplitude difference between the CP and CG horizontal positions and, consequently, the higher the (CP−CG)/CP trajectory path length ratio. In other words, this ratio can be interpreted as an indicator of the resultant stiffness, which is defined by the active stiffness controlled by the muscles and the passive stiffness of the elastic properties of the activated muscles, joints, and adjacent tissues in a given position.

Therefore, the different (CP−CG)/CP ratios observed between the subjects could be due in part to the inclination of their body, sometimes called the load stiffness. As presented above, an inclination of the body increases both the active and passive stiffness. This load stiffness can be indirectly estimated by the inclination of the body or the horizontal distance between the heel and the CG. The absence of significant difference of the average CG positions between the two conditions (Figure 6) indicates that the load stiffness was relatively constant for each subject between the two conditions. For example, the modification of the average CG positions between the two conditions studied (EO and
EC) corresponds approximately to a modification of the inclination of 0.2° for subject 4 and 0.04° for subject 5. These modifications of the inclination are too small to substantially modify the load stiffness. Therefore, the modifications of the (CP – CG)/CP ratio observed for these 2 subjects between the two conditions indicate that they have increased their muscular stiffness in the EC condition, resulting in a modification of the subject’s reflex response, probably to counteract the effect of the visual deprivation. In other words, when the load stiffness remains quite constant, this ratio allows the detection of a modification of the subject’s active stiffness between two sensory conditions (here between eyes open and eyes closed conditions) due to a different subject’s neuromuscular activity. This analysis can also be done with a shorter duration analysis to study the evolution of the subject’s active stiffness for a given condition. However, this ratio must be computed for a duration superior to 1.5 s to be applicable (Caron, 1997). Finally, it is important to note that the (CP – CG)/CP ratio used in the present study does not allow a quantitative determination of the contribution of passive versus active stiffness and the contribution of load versus reflex response stiffness.

The model used in the present study did not take into account the damping force (or moment) of the muscle system. This damping force depends on both a damping coefficient and the lengthening rate of the muscles. First, the damping coefficient decreases with the mean torque level (Hunter & Kearney, 1982). As it was presented above, the standing posture is characterized by a low level of force (or moment) developed by the muscles and resulting in a low damping coefficient. Finally, the muscular velocity is small. Therefore, it can be assumed that the system is underdamped.

Different models have been used to evaluate the active and passive stiffness. Fitzpatrick, Taylor, and McCloskey, (1992) expressed the relationship between ankle torque and ankle angle as a load stiffness to evaluate the ankle stiffness of standing humans in response to imperceptible perturbation. Winter, Patla, and Prince (1997) have suggested that the control mechanism would be a stiffness control. This evaluation of stiffness was based on the squared mean power frequency of the CP – CG horizontal position differences. This mean power frequency did not take into account the entire bandwidth of the signal (see Figure 2), and the accuracy of the method depends directly on the trial’s duration. Davis and Grabiner (1996) used the inverted pendulum model, the spring mass system, and the measurement of the CP motion to evaluate the joint and muscle stiffness in single-limb postural control. The (CP – CG)/CP ratio used in the present study seems to provide a new possibility to evaluate a modification of the net active stiffness in standing posture. This net stiffness takes into account the stiffness of the entire muscles activated by the subjects. This possibility was discussed only in consideration of the inverted pendulum model, which appears to be relevant for healthy subjects standing on a force plate (Winter, Prince, & Patla, 1997). Some authors have proposed a flexible pendulum model of quiet standing (Chow & Collins, 1995; Lauk et al., 1999). This model was also used to estimate muscle stiffness in subjects with Parkinson’s disease (Lauk et al., 1999). However, using a model represented by a multi-link inverted pendulum does not induce a modification of the moment of inertia of the body during quiet standing. Therefore, the amplitude CG/CP relationship in the frequency domain and the (CP – CG)/CP ratio used in the present study could also be available. Nevertheless, this model raises the representation of the CG motion as the net performance in quiet standing. This question remains a subject for future investigations.

In conclusion, the differences observed between the net neuromuscular control and the net performance concerning the trajectory path length parameter clearly show that these two variables must be analyzed simultaneously. These differences also confirm that
the CP variable is insufficient to evaluate the net performance. The simultaneous analysis of these two variables is facilitated by the simple and accurate CG estimation method presented here. Their comparison depends on the active and passive stiffness, and allows a comparison of the subjects’ reflex response between two sensory conditions. This estimation has to be compared to other models presented above; however, this comparative analysis appears helpful in understanding the control of balance.

References


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