Identification of Release Conditions and Aerodynamic Forces in Pitched-Baseball Trajectories

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Pitched-baseball trajectories were measured in three dimensions during competitions at the 1996 Summer Olympic games using two high-speed video cameras and standard DLT techniques. A dynamic model of baseball flight including aerodynamic drag and Magnus lift forces was used to simulate trajectories. This simulation together with the measured trajectory position data constituted the components of an estimation scheme to determine 8 of the 9 release conditions (3 components each of velocity, position, and angular velocity) as well as the mean drag coefficient $C_D$ and terminal conditions at home plate. The average pitch loses 5% of its initial velocity during flight. The dependence of estimated drag coefficient on Reynolds number hints at the possibility of the drag crisis occurring in pitched baseballs. Such data may be used to quantify a pitcher’s performance (including fastball speed and amount of curve-ball break) and its improvement or degradation over time. It may also be used to understand the effects of release parameters on baseball trajectories.

Key Words: baseball, aerodynamics, pitching, estimation

Introduction

Experimental measurements in baseball span more than 50 years. The first accurately recorded fastball (98.6 mph or 44.1 m/s) was thrown by Bob Feller in 1946 (Quigley, 1984), but peak speeds have not increased significantly since then. In 1949 Sikorsky and Lightfoot became the first investigators to measure the lift force on a baseball using a wind-tunnel (Alaways & Lightfoot, 2001; Drury, 1953). Major-league baseballs were mounted to a small electric motor and rotated at speeds ranging from 0 to 1200 rpm, clockwise and counter-clockwise, at wind stream speeds of 35.8, 42.5, and 49.2 m/s (80, 95, and 110 mph). The lift force was measured for the four-seam and two-seam orientation. The results showed that seam orientation plays a major role in the lift force.

In the same year, Davies (1949) measured the aerodynamic forces on golf balls in a wind tunnel. Briggs (1959) essentially repeated the experiment of Davies but with baseballs at spin rates up to 1800 rpm and speeds of 54.7 m/s (122 mph). Briggs used balls that were spinning about a vertical axis and thus gave the maximum lateral deflection, whereas in Davies’ measurements the axis of rotation was horizontal and normal to the wind stream. Briggs concluded that the lateral deflection was proportional to $\omega V^2$. Later Watts and Ferrer (1987) also measured lift on spinning baseballs in a wind tunnel but at speeds less...
than 17.9 m/s (40 mph). They concluded that, at these speeds, lift was proportional to $\omega V$ rather than $\omega V^2$ and was not dependent on seam orientation.

The purpose of this paper is twofold. First, we explain a new model-based smoothing technique for the estimation of baseball release conditions. In addition, using the technique, we quantify the performance of elite pitchers at the 1996 Olympic games, including the initial and final velocities, average drag coefficients, vertical and lateral break, and estimates of spin.

**Methods**

Pitched baseball trajectories were measured in three dimensions during competitions at the 1996 Summer Olympic games using two high speed video cameras and standard DLT techniques. A dynamic model of baseball flight and this experimental trajectory data constitute the inputs to an estimation scheme that determines initial conditions (three components each of velocity and position, and two of angular velocity) as well as the mean drag coefficient $C_D$ and terminal conditions at home plate. Because this model-based procedure allows the estimation of the initial conditions, it alleviates the necessity to differentiate or smooth noisy position data. The results of such an estimation procedure may be used to quantify the pitcher’s performance (e.g., fastball speed and amount of curve ball break) and its improvement or degradation over time, the efficacy of umpires in their judgment of balls and strikes, and even the initial velocity of batted balls.

To characterize the position $\mathbf{r} = (x, y, z)$ of the center of mass (c.m.) of the ball at any instant, we used an orthogonal, right-handed coordinate system (unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$) with its origin at the apex (rear-most point) of home plate and with the $xy$ plane horizontal. The $xz$

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**Figure 1** — Definitions of angular velocity vector and aerodynamic forces: (a) The orientation of the angular velocity vector $\omega$ with spin magnitude, $\omega_s$, is specified by two angles $\theta$ and $\phi$. (b) The total aerodynamic force has components parallel (drag $D$) and perpendicular to the relative wind. The perpendicular component is further decomposed into components perpendicular (lift $L$) and parallel (side force $Y$) to the angular velocity vector.
plane contains the center of the pitching rubber [the center of the top surface of which is located at coordinates (18.52, 0, 0.25) m], the y-axis points generally in the third-base direction, and z is directly upward. Thus, right-handed pitches are typically released with initial positions near r = (17, 1, 1.5) m. The initial velocity of all pitches have magnitudes between 29 and 45 m/s (65 and 100 mph) and are pointed mostly in the negative x direction.

The spin (or angular velocity) ω relative to inertial space of a typical baseball in flight (Figure 1) is defined by its magnitude ω, elevation φ (the angle between ω and the horizontal plane) and azimuth θ (the angle between the x axis and the projection of ω in the horizontal plane). Thus the xyz components of the angular velocity are given by

\[ \omega = (\omega \cos \theta \cos \phi, \omega \sin \theta \cos \phi, \omega \sin \phi) \]  

(1)

Bodies moving in motionless fluids are affected by pressure and viscous shear stresses acting on the surface. The resultant drag force acting parallel to the velocity \( V \) is given by the expression

\[ D = -\frac{\rho C_D A |V|^2}{2} \]  

(2)

where \( A = \pi r^2 \) is the frontal area, \( r = d/2 \) is the ball radius, \( \rho \) is air density and \( C_D \) is the dimensionless drag coefficient. \( C_D \) is usually determined experimentally using wind tunnel tests (Hoerner, 1965; Roberson & Crowe, 1980) and is found to be a strong function of the Reynolds number,

\[ \text{Re} = \frac{\rho V d}{\mu} \]  

(3)

where \( V \) is the speed of the ball and \( \mu \) is the dynamic viscosity of air.

Figure 2 — Drag coefficient \( C_D \) versus Reynolds number for spheres of varying roughness (after Achenbach, 1974; Bearman & Harvey, 1976). See also Frohlich (1984).
C_D is also a function of the surface roughness of the ball. The dependence of C_D on Re (Figure 2) for spheres of varying roughness was investigated by Achenbach (1974) and, for much rougher golf balls, by Bearman and Harvey (1976). The drag coefficient C_D is nearly constant at a value between 0.4–0.5 over a range of Reynolds numbers from $10^3$ to slightly above $10^5$ (corresponding to speeds in air of from 0.213 to 21.3 m/s) and then, at a Reynolds number which depends on surface roughness, abruptly decreases by a factor of between 3 and 5. For an absolutely smooth sphere, this transition occurs near $Re = 3.5 \times 10^5$. However, for rough spheres, the transition occurs at considerably lower Reynolds numbers. This phenomenon, known as the drag crisis, has been discussed by Achenbach and Frohlich (1984). Frohlich calculated its effects on baseball trajectories and speculated on an equivalent roughness for the baseball. Since, as will be shown below, the translational velocity changes by only about 5% during a pitch, and the angular velocity changes by even less (Alaways, 1998; Selin, 1957), we assume that the drag coefficient is constant but different, in general, for each pitch. This is not to say that there are no effects of spin and speed on drag, but only that during a given pitch a constant drag coefficient is a good approximation.

That force component perpendicular to the velocity, termed lift, is given (Alaways, 1998) by an expression similar to (2):

$$L = \frac{\rho C_L A V^2}{2} \frac{\omega \times V}{|\omega \times V|}.$$  \hspace{1cm} (4)

The dimensionless lift coefficient, $C_L$, is only a weak function of Re, but it depends as well on the spin parameter $S = r \omega / V$ and on the roughness of the ball.

![Figure 3 — $C_L$ versus spin parameter $r \omega / V$. The dashed-line average of the two- and four-seam data of Sikorsky and Lightfoot was used in the model.](image)
There is a strong dependence of $C_L$ on spin parameter for baseballs (Figure 3). The curved lines are the two-seam and four-seam experimental data (two- and four-seam curve balls are defined by the number of seams on the ball that trip the boundary layer at the ball’s surface during rotation) measured by Sikorsky and Lightfoot (Alaways & Lightfoot, 2001). For spin parameters less than 0.1, the four-seam lift coefficient is between two and three times larger than $C_L$ for the two-seam orientation. Because there was no way to determine experimentally the seam orientation of the pitches, the dashed line $C_L = 1.5 \frac{r\omega}{V}$ was used. This is consistent with the assumption of Watts and Bahill (1990) (based on the measurements of Watts & Ferrer, 1987) that $C_L$ depends almost linearly on spin parameter. All estimated spin parameters were less than 0.25 so that the linear approximation provided an average of the two experimental curves.

The estimation scheme described below operates loosely as follows. The curvature of the trajectory is determined by matching it to the experimental data and is attributed to a combination of gravitational and aerodynamic forces. Thus, the forces will be known relatively accurately (the accuracy will be discussed quantitatively below). However, because of the uncertainty in the relationship between the spin parameter and the lift force coefficient (Figure 3), due to unknown seam orientation and unknown aerodynamic force characteristics of Olympic baseballs, it will not be possible to deduce the spin precisely, even though the forces will be determined accurately. The use of the average linear approximating curve (Figure 3) can therefore result in errors in spin rate as large as 100%.

Shear stresses on the surface of a spinning ball also cause a torque about the c.m., but this causes less than a 1% change in spin over the roughly 0.45 s flight (Alaways, 1998). Thus we assume spin is constant in magnitude and direction.

The translational equations of motion are

$$m\ddot{r} = L + D - mgk$$  \hspace{1cm} (5)

where the vector $r$ consists of the three Cartesian coordinates $[x y z]$, $m = 0.150$ kg is the ball mass, and $g = 9.81$ m/s$^2$. Integration of the equations of motion given initial conditions produces a sequence of positions of the ball.

Two 120-Hz video cameras (resolution $640 \times 480$ pixels) were placed on over-hanging camera platforms on the front lip of the second level of Atlanta’s Fulton County Stadium. Camera A was placed directly behind home plate and camera B near the vertical plane containing second and third bases. The cameras were zoomed so that the pitcher-catcher axis occupied nearly all of the respective fields of view. Direct linear transformation (DLT) calibration later determined the A and B camera locations in the $xyz$ coordinate system to be $(–41.901, 2.139, 15.597)$ m and $(–23.810, 56.075, 15.890)$ m, respectively. From these relatively large distances, the pitched ball occupied only one or two pixels of the video frame and, although clearly visible to the eye, had sufficiently low contrast with the grass and dirt backgrounds to prevent automatic digitization. Twenty-one randomly selected pitches (10, 5, and 6 from the first, fifth, and sixth innings, respectively, of the Cuba-Japan game in the preliminary round) were chosen for manual digitization and analysis.

Camera calibration used a skeletal, cubic, tubular steel, calibration frame $(1.5 \times 1.5 \times 1.5$ m) on which were mounted 29 individually switchable incandescent spherical lights. The positions of the lights were determined (to within approximately 0.001 m) by surveying with a Nikon D-50 theodolite. This cube was adjustably mounted on a bearing about a vertical axis and supported on a three-legged triangular base. The rear leg of the base, located under the bearing, could be positioned (within 0.001 m) on either the rear point of home plate or the center of the pitching rubber. The front two legs of the base were threaded,
vertically aligned bolts that allowed leveling (within approximately 0.0005 rad) of the base and cube on both the irregular mound and home plate area. Finally, using horizontal lag bolts, the cube could be rotated about the vertical bearing so that it could be aligned in yaw (to within 0.0001 rad) using a spotting telescope.

Incandescent lights were chosen as calibration markers, rather than more common passive spherical ones, because of the large distances between the cameras and the cube, the relatively wide fields of view, and the uncontrolled lighting conditions under which calibration was required. Two successive camera calibration views of the sequentially switched lights were taken, with the cube positioned at the mound and at home plate, respectively. The known positions of the lights on the cube and of the cube on the field were used together with a standard DLT calibration scheme to locate the cameras and determine camera parameters. These parameters were used with synchronized 120-Hz images (Abdel-Aziz & Karara, 1971) to calculate ball position at N frames. We denote by the vectors $\mathbf{x}^m$, $\mathbf{y}^m$, and $\mathbf{z}^m$ the N vectors of measured values of $x$, $y$, and $z$, respectively.

As discussed above, the ball’s trajectory is completely determined by integrating the equations of motion, given nine initial conditions. Several possible techniques exist for the recovery of these release conditions from experimental data. First (or higher) order difference schemes allow estimation of derivative data from positions, but these techniques are susceptible to the amplification of the experimental noise. Various smoothing schemes have been proposed to ameliorate this noise amplification, but none is entirely satisfactory.

In this case the position quantization errors are large. With a field of view encompassing the entire 20-m flight of the ball, the length of one camera pixel corresponded to approximately 4 cm, more than half the ball diameter. During manual digitization, it was difficult to determine ball position with an accuracy better than one pixel, partly due to the quantization just discussed but also because of the varying contrast between the ball and the grass and dirt field surface.

Given this relatively noisy data, a technique much better than smoothing relies on an assumed theoretical dynamic model for flight. It asks and answers the question: What is the set of release conditions that, when used as initial conditions for the theoretical model, best predicts the experimental data, including not only that part of the data near the release point but also the position data for the entire flight? Indeed, we find that the data near the end of the trajectory is more important in the determination of accurate estimates for lift and drag than is the data near release.

The trajectory of the ball depends uniquely on its position, translational velocity, and angular velocity at the instant of release. Therefore the $i$th position of the center-of-mass of the ball can be written in functional form as:

$$\begin{bmatrix} x_i^p, y_i^p, z_i^p \end{bmatrix} = \begin{bmatrix} x_i^m, y_i^m, z_i^m \end{bmatrix} p$$

where $x^p$, $y^p$, and $z^p$ are predicted values of $x$, $y$, and $z$ using estimated initial conditions and the parameter vector $p = (x_0^o, y_0^o, z_0^o, \dot{x}_0^o, \dot{y}_0^o, \dot{z}_0^o, \phi, \omega_0^o, C_D)$ is the $m = 9$ vector of initial conditions and the drag coefficient.

Note that $p$ does not include the azimuth angle $\theta$. As seen from Equation (4), the $x$ component $\omega_c$ crossed into the velocity vector $\mathbf{V}$ (which is nearly all in the $x$ direction) is nearly zero. Because the associated Magnus forces and resulting deviations of the trajectory are small, it is not possible to estimate accurately the $x$ component of $\omega$. We therefore arbitrarily assume that the azimuth angle $\theta$ is $\pi/2$ in all cases (typical of a normal fastball).

Define the performance index, $R$, as the residual sum of squares of position errors in all three directions, the differences between the measured and predicted positions,
where \( x^p \), \( y^p \), and \( z^p \) are the N vectors of predicted values of \( x \), \( y \), and \( z \), respectively, \( x^m \), \( y^m \), and \( z^m \) are the N vectors of measured values of \( x \), \( y \), and \( z \), respectively, N is the total number of frames in the experimental data set for the center-of-mass trajectory, and superscript T denotes the matrix transpose. The minimization of \( R \) through the choice of the initial conditions (parameters) is then a classical nonlinear least-squares estimation problem (Hubbard & Alaways, 1989). The necessary condition for the optimality of the least-squares estimate of the parameters, \( p^* \), is that the gradient of \( R \) vanish when \( p = p^* \). In other words,

\[
g(p^*) = \left. \frac{\partial R}{\partial p} \right|_{p = p^*} = 0
\]

But by the chain rule,

\[
g(p^*) = \frac{\partial R}{\partial p} = \left( \frac{\partial x^p}{\partial p} \right)^T \frac{\partial R}{\partial x^p} + \left( \frac{\partial y^p}{\partial p} \right)^T \frac{\partial R}{\partial y^p} + \left( \frac{\partial z^p}{\partial p} \right)^T \frac{\partial R}{\partial z^p}
\]

Furthermore, since \( R \) is a sum of three quadratic forms, the right-hand factors in each term of Equation (9) are equal to the residual (error) vectors in \( x \), \( y \), and \( z \):

\[
\frac{\partial R}{\partial x^p} = (x^p - x^m), \quad \frac{\partial R}{\partial y^p} = (y^p - y^m), \quad \text{and} \quad \frac{\partial R}{\partial z^p} = (z^p - z^m)
\]

Notice that the left factors in each term of Equation (9) are just the three \( M \times N \) Jacobian matrices \( J_x, J_y, \) and \( J_z \) of partial derivatives with respect to \( p \) of \( x^p \), \( y^p \), and \( z^p \), respectively. These must be calculated at each iteration of the estimation algorithm by \( M + 1 \) simulations, perturbing each parameter in turn.

In what follows, the subscript \( k \) denotes a quantity evaluated at \( p_k \), which denotes the current estimate of the initial conditions. If we can evaluate the first and second derivatives of a general performance index \( R \) with respect to the parameters \( p \), then \( R \) can be approximated by a quadratic model consisting of the first three terms of the Taylor series expansion of \( R \) about the current estimate \( p_k \)

\[
R(p_k + \delta p) \approx R_k + g_k^T \delta p + \frac{1}{2} \delta p^T G_k \delta p
\]

The minimum value of the right-hand side of Equation (11) occurs (Gill et al., 1981) when \( \delta p \) satisfies the linear equation

\[
G_k \delta p = -g_k
\]

where \( G_k \) denotes the Hessian matrix of \( R \) evaluated at \( p_k \) and \( g_k \) is the gradient of \( R \) also evaluated at \( p_k \).

\[
g_k = \left. \frac{\partial R}{\partial p} \right|_{p = p_k}
\]

In the special case that occurs here where \( R(p) \) is a sum of squares of nonlinear functions of \( p \), the gradient and Hessian matrix have a special structure (Gill et al., 1981). The \( M \times M \) Hessian matrix \( G(p) \) is given by
\[ G(p_k) = J(p_k)^T J(p_k) + Q(p_k) \]

where \( Q(p_k) \) is as described in (Gill et al., 1981).

When \( R \) approaches zero as \( p_k \) approaches the least-squares solution, the matrix \( Q(p_k) \) also tends to zero. In this case the Newton direction \( \delta p_k \) can be approximated by the solution of the set of linear equations:

\[ J_k^T J_k \delta p_k = -J_k^T f_k \]

where \( f = [(x^p - x^M)^T (y^p - y^M)^T (z^p - z^M)^T]^T \)

The next estimate is found from

\[ p_{k+1} = p_k + \delta p_k. \]

This method is called the Gauss-Newton method. One of its advantages is that it requires the evaluation of only the \textit{first} derivatives of \( R \). In addition, solving Equation (15) using the singular-value decomposition (Gill et al., 1981) avoids unnecessary exacerbation of the ill-conditioning, a common feature of nonlinear least-squares problems of this type.

The numerical implementation of the estimation algorithm was straightforward, and there were no apparent problems encountered with convergence to local, rather than global, minima. Though not crucial in this problem, faster convergence can be obtained by using good initial guesses derived from the raw data.

The estimation procedure can be improved by implementing a univariate minimization of \( R \) in the \( \delta p_k \) direction using a modified Fibonacci search that not only returns the unimodal minimum over a search interval but also varies the search interval location to insure that the global minimum in the \( \delta p_k \) direction is found (Gill et al., 1981).

The estimation procedure consists of the following iterative algorithm:

1. Guess an initial parameter vector \( p \) and simulate.
2. Perturb, in turn, each entry of \( p \), holding the other entries at their nominal values, and simulate. This step will yield \( M \) simulations, where \( M \) is the number of parameters in the vector \( p \).
3. Using the \( M + 1 \) trajectories in steps 1 and 2 together with the measured trajectory, a Newton direction \( \delta p_k \) is calculated.
4. Using a univariate optimization, the optimal magnitude of \( \delta p_k \) is determined to minimize the residual in the \( \delta p_k \) direction.
5. Repeat steps 2, 3, and 4 using the updated \( p \) vector until the gradient of residual, \( g(p) \), is zero.

The most important advantage of the estimation scheme is that the noisy experimental kinematic data need not be differentiated or filtered in any way. In fact the estimated trajectory can be considered to be a filtered data set, since it passes through the experimental data in a least squares sense. It is also entirely consistent with the best available model for the trajectory dynamics (i.e., how the aerodynamic forces are generated). An additional advantage is that the iterative estimation scheme provides (Hubbard & Alaways, 1989) an estimate of parameter uncertainties at virtually no additional computational cost.

\textbf{Results}

The measured xy and xz projections for two typical pitches, c109 and c510, are compared with estimated trajectories (Figure 4). Optimal estimates for the initial conditions and drag coefficient for these two pitches resulted in an rms position error of only 1.8 and 1.2
Figure 4 — Overhead and side views of two typical pitches, c109 and c510; measured positions (circles), fitted curve from estimated initial conditions (solid line), theoretical spin-free trajectory (dashed line). The two components of the break, $b_1$ and $b_2$, are the differences between the fitted and spin-free trajectories at $x = 0$. 
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For all 21 pitches, the average rms error was 1.4 cm. The average uncertainty (Table 1) in each component of initial position (0.006 m) is smaller than the rms error in the fitted position by a factor of roughly 2.5. Also the errors in estimates of initial velocity are much smaller than one would expect using finite difference techniques. Note that Table 1 contains no uncertainty for the angle $\theta$.

Also plotted in Figure 4 are the spin-free (and thus lift-free) trajectories, clearly showing the break for the same two pitches. We define the two components of the break, $b_y$ and $b_z$, to be the differences, in $y$ and $z$, respectively, between the fitted and spin-free trajectories at $x = 0$, and the total break to be the rss of its two components, $b = (b_y^2 + b_z^2)^{1/2}$. Note that for the waist-high pitch c109 (Figure 4a), where $v_o = 35.9$ m/s, the break of 0.39 m is nearly all horizontal. Although the estimated initial spin $v_o = 18.5$ rev/s was substantial, the spin angular velocity vector was nearly vertically oriented ($\phi = 85^\circ$) and therefore the spin induced a nearly horizontal lift force. As a result, the deviation from the purely gravitational trajectory in the $z$ direction was small ($b_z = 0.03$ m). Contrast pitch c109 with pitch c510 (Figure 4b), a knee-high fast ball ($v_o = 42$ m/s), which had a larger initial spin $\omega_o = 27.3$ rev/s but which consisted of nearly equal parts of vertical spin and backspin ($\phi = -42^\circ$). In this case the vertical break of 0.37 m is even larger than the lateral break of 0.34 m. Both pitches crossed the plate in the strike zone as defined by the rules.

The next five figures show some overall results including data from all pitches analyzed. The initial speed estimate is plotted (Figure 5) versus the estimate of the vertical component of the initial velocity. Initial speeds ranged between 34.3 and 42.5 m/s (76.7 and 95.1 mph). Although there are some intermediate velocities, the pitches are seen to

![Figure 5 — Initial release speed versus initial vertical velocity. Pitches appear in two groups: fastballs and curves. Faster pitches have a larger initial downward velocity.](image)

<table>
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<th>Parameter</th>
<th>$x_o$</th>
<th>$y_o$</th>
<th>$z_o$</th>
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<th>$Vy_o$</th>
<th>$Vz_o$</th>
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<th>$\phi_o$</th>
<th>$\omega_o$</th>
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<td>rad</td>
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<td>—</td>
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<td>1.79</td>
<td>0.017</td>
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</table>

cm, respectively, only about half the quantization errors of 4 cm in measured positions. For all 21 pitches, the average rms error was 1.4 cm.
fall into two major groups, fastballs ($v_o$ about 42 m/s) and curves ($v_o$ about 36 m/s). The fastballs are released with substantially larger negative vertical velocities (about –2 m/s), while the more slowly thrown curve balls have more time to be acted on by gravity and thus require less initial downward velocity.

![Figure 6](image-url)  
**Figure 6** — Final speed (at $x = 0$) versus release speed. Pitches lose from 3 to 7% of their initial speed during the roughly 0.45 s flight.

![Figure 7](image-url)  
**Figure 7** — Total break versus estimated spin. Larger spin induces larger Magnus forces and thus causes a larger break. Because speed varies only by 10–20%, total break is nearly entirely accounted for by changes in spin.
Final speed (at \(x = 0\)) is considerably less than initial speed (Figure 6). The two main velocity groupings are still apparent, but the other feature evident is that each pitch loses roughly from 3 to 7% of its initial speed during the flight (a linear fit to the data which also passes through the origin has a slope of 0.95). The larger speed losses of about 6% are consistent with hand calculations. These show that a 0.15 kg ball released with an
initial velocity of 40 m/s and a drag coefficient of $C_D = 0.3$ experiences a drag force of about 1.1 N, a deceleration of about 7 m/s$^2$ (0.7 g’s) and a flight time of about 0.42 s.

Total break $b$ is a function of the estimated spin magnitude (Figure 7). A linear dependence would be expected since lateral force is assumed to be proportional to spin through the spin parameter and the assumed lift force function. The pitch with the most curved trajectory had total break of 0.62 m (2.03 ft).

The estimated drag coefficient $C_D$ is a function of the speed ratio $v_f / v_o$ (the ratio of final speed to initial speed; Figure 8). The drag coefficient varies by more than a factor of two over the range 0.16 to 0.34 and, as expected, is nearly linearly correlated with the speed ratio, since larger drag results in more speed loss.

Perhaps the most interesting result is the dependence of $C_D$ on initial Reynolds number (Figure 9). Although the number of data points is not large enough to be certain, there is certainly a hint of a functional relationship between $C_D$ and Re, which exhibits an abrupt decrease as in the drag crisis relationship discussed previously (Figure 2). That is, $C_D$ appears to drop suddenly after a Reynolds number of 160,000 to as low as 0.15 before rising again to 0.2 when the Reynolds number reaches 190,000. The drag model in this paper assumed a constant drag coefficient in spite of speed variations of about 5%, corresponding to decreases in Reynolds number of about 8,500. A more complex model might be used to identify the dependence of $C_D$ on Reynolds number by including a variable $C_D$.

An attempt to apply this kind of model will be made in the future.

**Discussion**

The method presented relies on experimental kinematic data of baseball flight to estimate the initial conditions at release as well as the coefficient of drag. Rather than differentiating the noisy data to obtain even noisier estimates (e.g., of initial velocities), the method instead relies on computer simulation and iterative refinement of the initial conditions in the simulation to produce a best fit to the experimental data. Even with camera distances of 45 and 60 m from the pitch, estimates of initial conditions yield simulated trajectories, with rms errors from the measured data of only 1.5 cm.

Although there are few results in the literature to compare with the results of this study, the initial conditions determined above do agree reasonably well with some previous reports. The speeds estimated here (34–42 m/s) correlate well with those measured by Selin (1957) and reported by Quigley (1984). The maximum break of 0.64 m estimated here is considerably larger than the 0.46 m break measured by Selin for collegiate pitchers and the 0.44 m predicted by Briggs (1959).

The method detailed here does have some limitations. The main one of these is its inability to calculate accurately the spin of the ball relying only on measured trajectories of its center. This has been discussed above and is due to uncertainties in the relationship between spin parameter and Magnus lift coefficient. This is inherent in any method which does not measure motion of more than one marker on the ball surface from which to infer rotation.

A possible important implication of the results presented above is that the drag crisis, first suggested by Frohlich (1984) to have a potentially large effect on the flight of the baseball, is somewhat corroborated by the data herein. Although further studies are warranted, it appears that there may be a minimum of the drag coefficient as a function of Reynolds number. The present paper is no more than a first step in investigating this phenomenon, although it does present an approach that might be fruitful in the future.

The method described in this paper, or a suitable modification of it, should be applicable to different sports or other instances of kinematic data processing. Potential applications are numerous. One example might be the estimation of post-impact conditions in spiking
a volleyball. Moreover, the use of a physically causal model (“model based smoothing”) should always be preferred, when available, to pure fitting or regression. This is because models consist of relationships containing physically meaningful parameters that we have reason to believe are descriptive of the physics. The models therefore have the potential to explain more complex behavior than polynomials or other more simplistic representations.

A simple polynomial regression to the data might produce nearly as good a fit and allow calculation of initial conditions. The approach taken in this paper, which is based on a physically plausible causal model using traditional aerodynamic relationships, allows not only the estimation of initial conditions in the pitches considered here, but also estimates of aerodynamic coefficients. Furthermore, the model can be used in an independent context, to investigate the effects of changes in release conditions and even changes in aerodynamic parameters, on pitch trajectories in general.

References


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